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MATHEMATICS
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PAPER 3 – DISCRETE MATHEMATICS

Thursday 15 May 2014 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

4 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 10]*

The weighted graph K, representing the travelling costs between five customers, has the following adjacency table.

	A	B	C	D	E
A	0	1	6	7	4
B	1	0	9	8	10
C	6	9	0	11	3
D	7	8	11	0	12
E	4	10	3	12	0

(a) Draw the graph K. *[2]*

(b) Starting from customer D, use the nearest-neighbour algorithm, to determine an upper bound to the travelling salesman problem for K. *[4]*

(c) By removing customer A, use the method of vertex deletion, to determine a lower bound to the travelling salesman problem for K. *[4]*

2. *[Maximum mark: 23]*

(a) Consider the integers $a = 871$ and $b = 1157$, given in base 10.

(i) Express a and b in base 13.

(ii) Hence show that $\gcd(a, b) = 13$. *[7]*

(b) A list L contains $n+1$ distinct positive integers. Prove that at least two members of L leave the same remainder on division by n. *[4]*

(c) Consider the simultaneous equations

$$\begin{aligned} 4x + y + 5z &= a \\ 2x + z &= b \\ 3x + 2y + 4z &= c \end{aligned}$$

where $x, y, z \in \mathbb{Z}$.

(i) Show that 7 divides $2a + b - c$.

(ii) Given that $a = 3$, $b = 0$ and $c = -1$, find the solution to the system of equations modulo 2. *[6]*

(d) Consider the set P of numbers of the form $n^2 - n + 41$, $n \in \mathbb{N}$.

(i) Prove that all elements of P are odd.

(ii) List the first six elements of P for $n = 0, 1, 2, 3, 4, 5$.

(iii) Show that not all elements of P are prime. *[6]*

3. *[Maximum mark: 10]*

(a) Draw a spanning tree for

(i) the complete graph, K_4;

(ii) the complete bipartite graph, $K_{4,4}$. *[2]*

(b) Prove that a simple connected graph with n vertices, where $n > 1$, must have two vertices of the same degree. *[3]*

(c) Prove that every simple connected graph has at least one spanning tree. *[5]*

4. *[Maximum mark: 17]*

(a) (i) Write down the general solution of the recurrence relation $u_n + 2u_{n-1} = 0$, $n \geq 1$.

(ii) Find a particular solution of the recurrence relation $u_n + 2u_{n-1} = 3n - 2$, $n \geq 1$, in the form $u_n = An + B$, where $A, B \in \mathbb{Z}$.

(iii) Hence, find the solution to $u_n + 2u_{n-1} = 3n - 2$, $n \geq 1$ where $u_1 = 7$. *[10]*

(b) Find the solution of the recurrence relation $u_n = 2u_{n-1} - 2u_{n-2}$, $n \geq 2$, where $u_0 = 2$, $u_1 = 2$. Express your solution in the form $2^{f(n)} \cos\left(g(n)\pi\right)$, where the functions f and g map $\mathbb{N}$ to $\mathbb{R}$. *[7]*